Edexcel GCE
Statistics S2
Advanced/Advanced Subsidiary
Wednesday 21 January 2009 – Afternoon
Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper. Answer ALL the questions. You must write your answer for each question in the space following the question. Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates
A booklet ‘Mathematical Formulae and Statistical Tables’ is provided. Full marks may be obtained for answers to ALL questions. The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2). There are 7 questions in this question paper. The total mark for this paper is 75. There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled. You should show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

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Total

Turn over
1. A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

(a) more than 2 daisies,  

(b) either 5 or 6 daisies.

The botanist decides to count the number of daisies, \( x \), in each of 80 randomly selected squares within the field. The results are summarised below

\[ \sum x = 295 \quad \sum x^2 = 1386 \]

(c) Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places.

(d) Explain how the answers from part (c) support the choice of a Poisson distribution as a model.

(e) Using your mean from part (c), estimate the probability that exactly 4 daisies will be found in a randomly selected square.
Question 1 continued

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Question 1 continued
2. The continuous random variable $X$ is uniformly distributed over the interval $[-2, 7]$.

(a) Write down fully the probability density function $f(x)$ of $X$. 

(b) Sketch the probability density function $f(x)$ of $X$.

(c) Find $E(X^2)$.

(d) $P(-0.2 < X < 0.6)$. 

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3. A single observation $x$ is to be taken from a Binomial distribution $B(20, p)$. This observation is used to test $H_0: p = 0.3$ against $H_1: p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be as close as possible to 2.5%.

(b) State the actual significance level of this test.

The actual value of $x$ obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.
Question 3 continued
4. The length of a telephone call made to a company is denoted by the continuous random variable $T$. It is modelled by the probability density function

$$f(t) = \begin{cases} 
kt & 0 \leq t \leq 10 \\
0 & \text{otherwise}
\end{cases}$$

(a) Show that the value of $k$ is $\frac{1}{50}$

(b) Find $P(T > 6)$.

(c) Calculate an exact value for $E(T)$ and for $\text{Var}(T)$.

(d) Write down the mode of the distribution of $T$.

(e) Sketch the graph of a more suitable probability density function for $T$. 

It is suggested that the probability density function, $f(t)$, is not a good model for $T$. 

(1) 

(2) 

(3) 

(4) 

(5) 

(1)
Question 4 continued

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(Total 12 marks)
5. A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.

(b) Find the probability that there are at least 2 defective components in the box.

(c) Using a suitable approximation, find the probability that a batch of 250 components contains between 1 and 4 (inclusive) defective components.
Question 5 continued

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6. A web server is visited on weekdays, at a rate of 7 visits per minute. In a random one minute on a Saturday the web server is visited 10 times.

(a) (i) Test, at the 10% level of significance, whether or not there is evidence that the rate of visits is greater on a Saturday than on weekdays. State your hypotheses clearly.

(ii) State the minimum number of visits required to obtain a significant result.

(b) State an assumption that has been made about the visits to the server.

In a random two minute period on a Saturday the web server is visited 20 times.

(c) Using a suitable approximation, test at the 10% level of significance, whether or not the rate of visits is greater on a Saturday.
Question 6 continued

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(Total 14 marks)
7. A random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \frac{2}{9}x + \frac{1}{9} & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the cumulative distribution function $F(x)$ can be written in the form $ax^2 + bx + c$, for $1 \leq x \leq 4$ where $a$, $b$, and $c$ are constants.

(b) Define fully the cumulative distribution function $F(x)$.

(c) Show that the upper quartile of $X$ is 2.5 and find the lower quartile.

Given that the median of $X$ is 1.88

(d) describe the skewness of the distribution. Give a reason for your answer.
Question 7 continued

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